DIVISIBILITY BY PRIME NUMBERS.

BY PROF. D. M. SENSENIG, NORMAL SCHOOL, INDIANA, PA.

Theorem. A prime number p, other than 2 and 5, is a divisor of a number N, if it is a divisor of the sum of its digits, taken in groups of as many figures each, as there are figures in the repetend produced from the fraction $1 \div p$.

Illustration. Thus 37 is a divisor of 61210122865, since it is a divisor of $865 + 122 + 210 + 61$, or 1258. Notice that $1 \div 37$ yields a repetend of three figures.

Demonstration. Put r for the number of figures in the repetend produced from $1 \div p$, t for the number of groups of r figures each in the number, and $n_1, n_2, n_3, \ldots\ldots n_{t-1}, n_t$ for the number of units in the successive periods, beginning at the right; then

$$N = n_t(10)^{tr} + n_{t-1}(10)^{(t-1)r} \ldots + n_3(10)^{3r} + n_2(10)^{2r} + n_1(10)^r.$$

But from the nature of repetends, p divides each of the terms $(10)^{tr}$, $(10)^{(t-1)r}$, $\ldots (10)^{3r}$, $(10)^{2r}$, and $(10)^r$, with a remainder of 1, hence it divides N with a remainder of $n_t + n_{t-1} + \ldots\ldots n_3 + n_2 + n_1 = R$. Now if R is divisible by p it is evident that p is a divisor of N; but R is the sum of the digits taken in groups of r figures together, hence p is a divisor of N, if it is a divisor of the sum of its digits taken in groups of r figures together.

COROLLARIES. 1. Three is a divisor of a number if it is a divisor of the sum of its digits taken singly, since $\frac{1}{3}$ produces a repetend of one figure.

2. Eleven is a divisor of a number if it is a divisor of the sum of its digits taken two together, since $\frac{1}{11}$ produces a repetend of two figures.

3. Thirty seven is a divisor of a number if it is a divisor of the sum of its digits taken three together, since $\frac{1}{37}$ produces a repetend of three figures.

4. One hundred and one is a divisor of a number if it is a divisor of the sum of its digits taken four together, since $\frac{1}{101}$ produces a repetend of four figures.

5. Forty one and two hundred and seventy one are divisors of a number if they are divisors of the sum of its digits taken five together, since $\frac{1}{41}$ and $\frac{1}{271}$ each produces a repetend of five figures.

6. Seven and thirteen are divisors of a number if they are divisors of the sum of its digits taken six together, since $\frac{1}{7}$ and $\frac{1}{13}$ each produces a repetend of six figures.

7. Any prime number that produces a perfect repetend is a divisor of a number if it is a divisor of the sum of its digits taken as many figures together as there are units in the prime number less one.

8. Since $\frac{n}{(999\ldots9)_r}$ is the general form of a common fraction producing a repetend of r figures, it follows that any divisor of $(999\ldots9)_r$ is a divisor of a number if it is a divisor of the sum of its digits taken r together.

NOTE. In a letter to the Editor, Prof. H. T. Eddy writes:—"In the very interesting Historical Sketch contained in your Sept. No. there is one omission which I feel should be supplied in the enumeration of the articles contributed to the Mathematical Monthly. I refer to Ferrel's investigations respecting the laws which govern atmospheric currents. These articles are regarded, I think, as the most important original investigations published in the Mathematical Monthly."

SOLUTIONS OF PROBLEMS IN NUMBER SIX, VOL. II.

SOLUTIONS of problems in No. 6, Vol. II, have been received as follows: From J. M. Arnold, 92; Prof. W. W. Beman, 93 and 94; Lieut. S. H. Baker U. S. N., 94; G. L. Dake, 92 and 95; G. M. Day, 97; Cadet E. S. Farrow, 92, 93, 94, 95 and 97; Henry Gunder, 92, 95 and 97; Christine Ladd, 93, 94 and 95; Artemas Martin, 92 and 95; Dr. A. B. Nelson, 92 and 95; O. D. Oathout, 92, and 97; Geo. H. Pegram, 95; K. M. Supten, 92 and 95; Prof. J. Scheffer, 92, 93, 94, 95 and 97; E. B. Seitz, 95 and 97; E. H. Westermann, 95; Prof. C. M. Woodward, 97.

92.—"A balloon is ascending vertically with a given velocity v, and a body is let fall from it, which touches the ground in t seconds; find the height of the balloon at the moment the body is let fall from it."

SOLUTION BY HENRY GUNDER, NORTH MANCHESTR, IND.

In t seconds a body will fall from *rest* $\frac{1}{2}gt^2$ feet. But from the conditions of the problem it ascends vt feet. Therefore it falls from a height of $(\frac{1}{2}gt—v)t$ feet.

93.—"To construct a triangle if the three radii of the circles, which touch the three sides externally, are given."

SOLUTION FURNISHED BY PROF. W. W. BEMAN, ANN ARBOR, MICH.

Christine Ladd writes: "The construction follows at once from the solution given by Chauvenet, 164, 12." Prof. Beman writes: "The following elegant solution of Problem 93 may be found – *in substance* – in Catalan's *'Theoremes et Problemes de Geometrie Elementaire'*, page 155: